ON PHYSICAL-INSPIRED SYNTHESIS OF SOUNDS

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Abstract. This small article will define a type of sound synthesis that takes inspiration from physical modeling but has important differences with it. The produced sounds exhibit *physical* characteristics that make them suitable for the sonification of real-world objects.

1. Introduction

The idea of producing sounds by copying the physical system that emits them dates back to the 80's and is directly related to subtractive synthesis. The first models (Karplus-Strong and extended Karplus-Strong) were, indeed, based on the application of special filters to noise-like sources in order to create a dissipation in the energy of high-frequency components, thus simulating the behaviour of real vibrating objects. In general, all these kind of models were centered on the idea that a vibrating object can be represented by means of two separate interacting entities: an *exciter* and a *resonator*. Typically, the exciter injects some energy into one or more resonators that, consequentially, give some energy back to the exciter creating a non-linear system with feedback. Figure 1 depicts the described concepts.

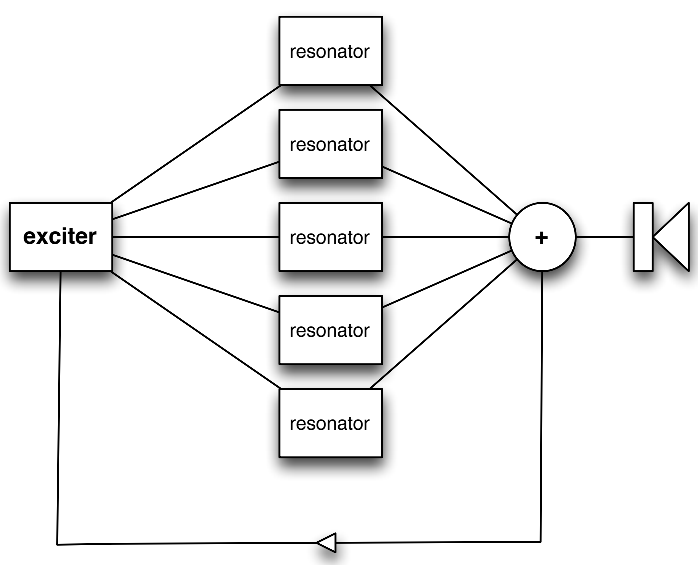

Figure 1. A typical exciter/resonators model.

In order to understand how this system can reproduce a physical vibrating object it is necessary to give some definition about mechanical vibrations.

2. Mechanical vibrations

A vibrating object can be represented by the *spring-mass* system, in which a mass is applied to a spring with a given stiffness constant. The temporal evolution of the created vibrations can be described by the following second-order homogeneous differential equation:

$$x = e^{-\alpha t} A cos(\omega_d t + \phi) \tag{1}$$

where:

- α is the *decay constant* of the system and depends on the mass and on the stiffness of the spring;
- ω_d is the natural angular frequency;
- A and ϕ are the respectively the amplitude and the phase of the vibration and are determined by the initial displacement and velocity.

Figure 2 represents a damped vibration as described by equation 1, also called *mode*. Despite the simplicity of the mass-spring model, complex systems can be analyzed in terms of independent sets of decaying modes.

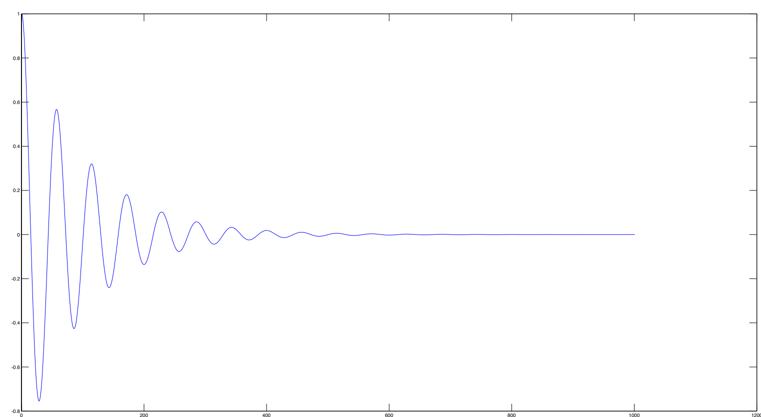

FIGURE 2. A damped vibration.

3. Modal synthesis and digital resonators

The complex dynamic behaviour of a vibrating object may be decomposed into contributions from a set of modes each of which oscillates at a single complex frequency; the generation of sounds using this approach is often called *modal synthesis*. An object that exhibits strong modes and is excited by striking or plucking is a good candidate for modal synthesis.

In the digital domain, the equation 1 can reproduced by means of the following second-order differential equation:

$$y = x \cdot b_0 - y \cdot z^{-1} \cdot a_1 - y \cdot z^{-2} \cdot a_2 \tag{2}$$

where z^{-n} is the delay of n digital samples, b_0, a_1 and a_2 are called coefficients and x is an input signal; the system described by equation 2 is usually called *two-poles* filter or digital *resonator* whose behavior is regulated by the value of the coefficients (figure 3 represents such a system).

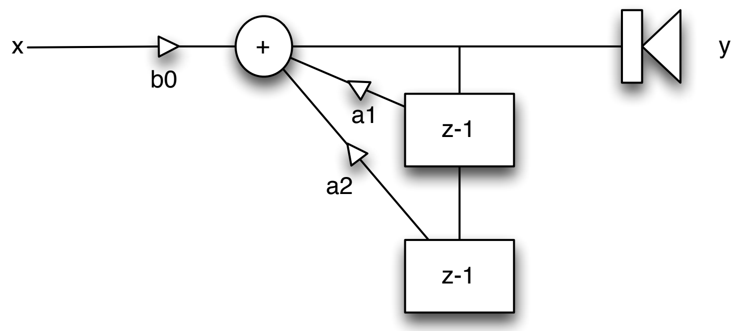

FIGURE 3. A two-poles filter.

A two-poles filter, indeed, can be designed to produce a peak at a specified frequency by setting its feedback coefficients as:

- $a_1 = -2 \cdot r \cdot cos(2 \cdot \pi \cdot f \cdot T_s)$
- $a_2 = r^2$

where r is the pole radius and T_s is the sampling period; the coefficient b_0 is consequentially computed to have a magnitude at the peak equal to 1.

A set of two-poles filters can be combined in parallel to simulate all the modes of a vibrating system; each resonator will have a different amplitude, center frequency, and rate of decay.

3.1. **Exicter/resonators interaction.** The estimation of the parameters for the resonators is a complex matter and is usually based of experimental measurements. While the frequencies and the decay rates are based on physical properties (such as *inharmonicity*), the **amplitudes are usually determined by the feedback interaction between the resonators and the exciter**.

If the exciter injects a digital impulse into the resonators, each of them will be equally excited and consequently all the amplitudes will be the same. On the other hand, if a feeback signal is added to the digital impulse, the excitation signal will exhibits a temporal smearing and a frequency equalization; for this reason each filter will react independently to the stimulus and will assume a different amplitude, thus generating a particular *timbre*. This interaction is normally regulated by a set of weights (called *modal weights*) that are multiplied to the individual output of each resonator and that derive either from wave equations or from experimental measurements.

4. PHYSICAL-INSPIRED SYNTHESIS

The simulation of a real vibrating object by means of modal synthesis (for example a musical instrument) can be a difficult task because of the complex analysis required for the estimation of the parameters. On the other hand, the simulation of a *likely*-physical instrument can be an interesting creative activity. With this kind of *physical-inspired* sound synthesis, infact, it could be possible to generate sounds that have special physical characteristics while not being generated by real vibrating objects.

What mainly differentiates a physical model from a physical-inspired model is the *missing* interaction between the exciter and the resonators. Instead, a *shaping function* is applied to exciter in order to create interesting evolving timbres as depicted in figure 4.

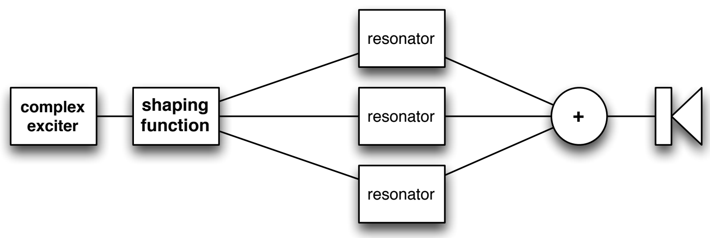

FIGURE 4. Physical-inspired synthesis.

The shaping of the exciter is performed through the selection of a complex stimulus (different from the digital impulse) and the multiplication of it with a set of coefficients (one for each resonator) that are similar, in functionality, to modal weights but are not derived from physical equations. Different criteria can be applied to create these weights, based on different perspectives; for example, it is possible, to use the laws used to calculate the frequencies also to handle the amplitudes.

4.1. **Models of shaping.** Among the various possible models for the shaping function, there are three that are particularly interesting:

companded harmonic: this model uses the companded series proposed by McAdams to create two related vectors for frequencies and amplitudes of each modes:

$$f_n = n^{\alpha} \cdot f_0 \tag{3}$$

where f_0 is the fundamental frequency of the sound, α is the *compantion* coefficient and n is the mode;

geometric series: this model propose a simple law for frequencies and amplitudes based on the geometric accumulation of values:

$$f_n = \beta \cdot f_{n-1} \tag{4}$$

where β is the geometric ratio;

model based: in this model the amplitudes and the frequencies are inferred from the spectral analysis of a target sample; the spectrum if first searched for peaks and then a spectral envelope is computed (figure 5 shows a typical envelope computed by means cepstral coefficients).

5. Conclusions

This short article showed the possibilities for sound synthesis created by the use of a physical-inspired approach. The central idea is based on the representation of a complex vibrating system by means of digital resonators whose amplitude is not

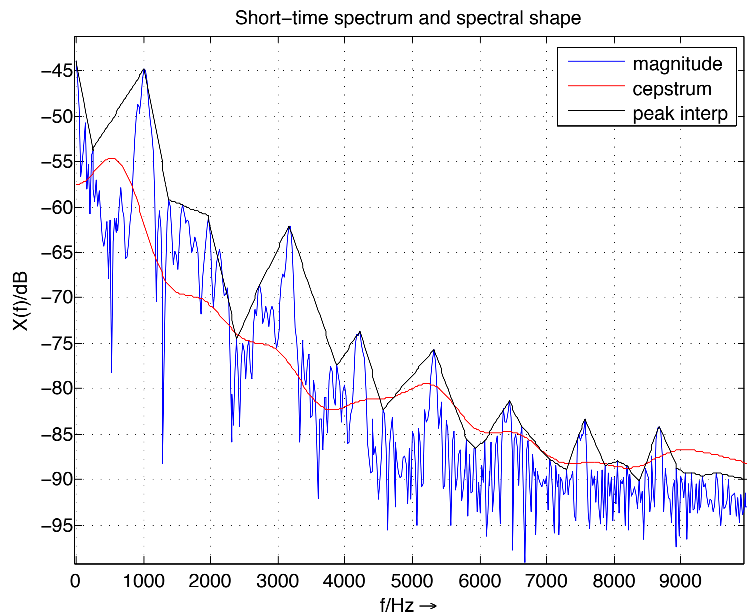

FIGURE 5. Spectral envelopes computed with cepstral coefficients.

given by wave equations but by different laws. The created sounds exhibit peculiar physical characteristics that make them suitable for the sonification of real-world objects; the described approach, moreover, is computationally affordable for real-time performances and can be easily implemented in dedicated languages such as Max/MSP.

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