Generalized series for spectral design

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1 Spectral design

The properties of convergency and divergency of mathematical series can be considered as atomic elements for spectral design. Using combinations of series it is possible to outline four basic spectral types:

1. convergent spectrum;
2. divergent spectrum;
3. convergent-divergent spectrum;
4. divergent-convergent spectrum.

Any combination of the four basic types can generate complex spectral patterns.

2 A generalized series

Two mathematical series are very important for music: the harmonic series is the base of the timbre of musical instruments, while geometric series encodes the frequency distrubution of the tempered tuning system for Western music.
With the unifications of these two important series it could possible to encode all harmonic spectra of musical instruments whose fundamental frequency lies in the tempered tuning system. For this reason, a new series G is defined in order to design the basic spectral types defined above. This new series is a generalization of the harmonic series (usually called *hyper-harmonic series* or *p-series*) and of the geometric series:

$$G = \sum_{n=1}^{\infty} \frac{1}{n^{\alpha} \cdot \beta^{n} + \gamma} \tag{2.1}$$

where α, β and γ coeffients are real numbers.
As an added benefit, also inharmonic spectra such as bell-like sounds can be described with equation 2.1.

2.1 Properties

Depending on the values of the coefficients, equation 2.1 can assume different morphologies:

- $\alpha = 1$, $\beta = 1$ and $\gamma = 0$ generates the harmonic series;
- $\alpha = 0$, $\beta > 1$ and $\gamma = 0$ generates the geometric series;
- $\alpha > 1$, $\beta = 1$ and $\gamma = 0$ generates the expanded-harmonic series;
- $\alpha < 1$, $\beta = 1$ and $\gamma = 0$ generates the compressed-harmonic series;
- for any value of α, if $\beta \neq 1$ and $\gamma = 0$ a geometric warping is applied to the harmonic series;
- for any value of β, if $\alpha \neq 1$ and $\gamma = 0$ an harmonic warping is applied to the geometric series;
- for any value of α and if β, if $\gamma \neq 0$ a translation is applied on the generated series.

2.2 MaxMSP implementation

The series implemented in MaxMSP as an expression; please note that this small patch must be used inside a *poly~*.

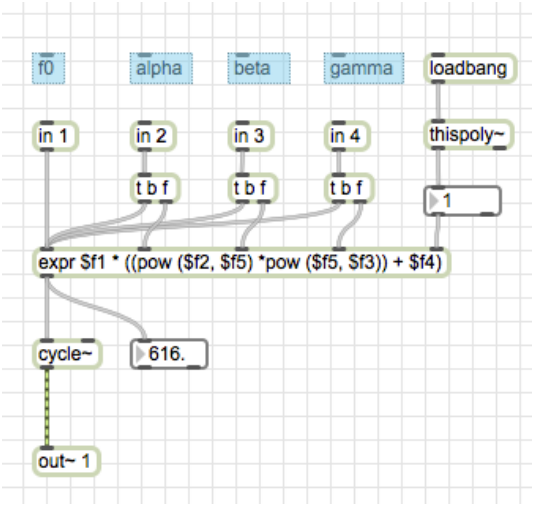

Figure 2.1: An implementation for MaxMSP